Experiment 9 — Kirchhoff's Voltage Law, Subscripted Voltages

EL 111 - DC Fundamentals

By: Walter Banzhaf, E.K. Smith, and Winfield Young
University of Hartford
Ward College of Technology

Objectives:

1. For the student to be able to measure the voltage across a circuit element, using a doubly-subscripted notation for that voltage.
2. For the student to assign a doubly-subscripted voltage to a circuit element.
3. For the student to write a Kirchhoff's Voltage Law equation for a circuit, using assigned polarities.

Equipment and parts:

Meters: Digital Multimeter (DMM)

Power Supply: Agilent E3631A DC power supply (0 to 20.0V DC)

Misc: Component Board

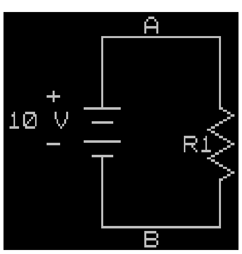
Resistors:

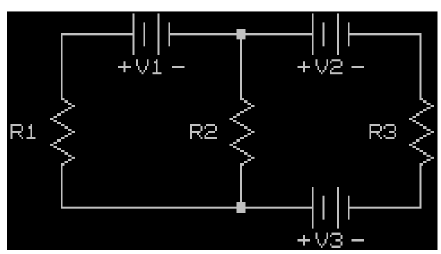
R#	R1	R2	R3	R4	R5	R6	R7	R8	R9
Ω	1.3k	4.7k	3.3k	10k	5.6k	1.3k	2k	1.3k	zero

Information:

Prof. Gustav Robert Kirchhoff (1824 - 1887) was a German pioneer in the study of electrical circuits. His pioneering work a century and a half ago is the basis of many of the analysis methods we use today, and a mastery of Kirchhoff's methods will help you in coursework you will have after DC Fundamentals. Let's be quite candid: you can't succeed in your courses without mastering Kirchhoff's laws.

At first glance, you may feel that this topic is silly, or unnecessary, since for many simple circuits the polarity of a voltage is obvious. For example, in the circuit on the left, below, it is totally obvious that the voltage across R1 is positive on top, negative on bottom. But, is it obvious what the polarity of voltage across resistor R2 is in the circuit on the right, below? Probably the answer is NO, since the polarity of that voltage depends on the relative size of the voltage sources, and the resistors, in the circuit.

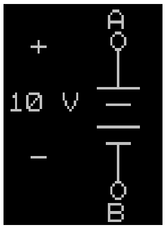




Procedure:

1. DOUBLY-SUBSCRIPTED VOLTAGES

1.a) Connect the "circuit" shown below. Notice it's not a very complicated circuit; it has ONE element, a 10 volt voltage source. Point **A** is the positive end of the battery, point **B** is the negative end.



1.b) Measure the voltage of this voltage source by connecting the RED lead of your DMM to point A, and the BLACK lead to point B. This is measuring V_{AB}, **the voltage at point A compared to the voltage at point B**. Record V_{AB} below:

V_{AB} = _______________

This result should tell us that "The voltage at point A is 10 volts more **positive** than the voltage at point B."

1.c) Measure the voltage by connecting the RED lead of your DMM to point B, and the BLACK lead to point A. This is measuring V_{BA}, **the voltage at point B compared to the voltage at point B**. Record V_{BA} below:

V_{BA} = _______________

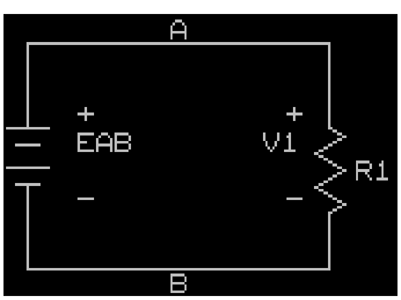
This result should tell us that "The voltage at point B is 10 volts more **negative** than the voltage at point A."

HELPFUL HINT: Notice that the **first subscript** (in V_{BA}, **B** is the first subscript) tells you where to put the **RED** lead of the voltmeter. It **DOESN'T** mean that point B is positive compared to point A; it just tells you how to connect the voltmeter. In 1.b), V_{AB} was positive; in 1.c), V_{BA} was negative. This is **the SAME voltage**; it's just a matter how we express it.

BELLY BUTTON ANALOGY: let your belly button be point **B**, and your toe be point **T**. If you are standing on your feet, the displacement between those two points can be expressed two ways:

displacement$_{BT}$ = height of belly button compared to toe. For many people, $d_{BT} \approx +3$ feet.

displacement$_{TB}$ = height of toe compared to belly button. For many people, $d_{TB} \approx -3$ feet.

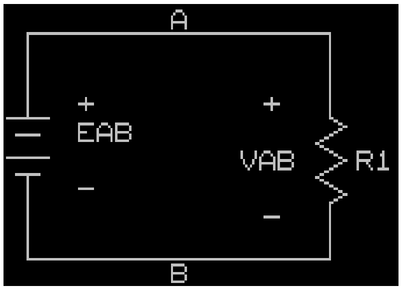


In the diagram to the left, E_{AB} is the voltage of the voltage source, expressed as "the voltage at node A compared to the voltage at node B." It's actually unnecessary to have the + and - signs shown, since E_{AB} says it all.

V1 is the voltage across resistor R1, BUT since it isn't a doubly-subscripted voltage, V1 **MUST** have the + and - signs to tell us how to measure V1 with a voltmeter.

1.d) Measure E_{AB} and record its value (*watch polarity*): E_{AB} = ____________

1.e) Measure V1 and record its value (*watch polarity*): V1 = ____________

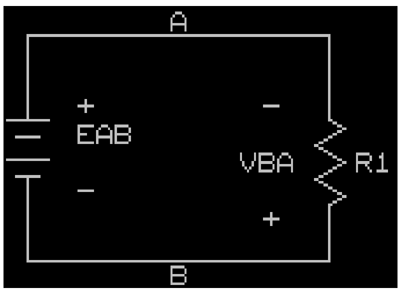


In the diagram to the left, E_{AB} is the voltage of the voltage source, expressed as “the voltage at node A compared to the voltage at node B.” It’s actually unnecessary to have the + and - signs shown, since E_{AB} says it all.

NOW, V_{AB} is the voltage across resistor R1, AND it is a doubly-subscripted voltage, so we don’t need the + and - signs to tell us how to measure V_{AB} with a voltmeter. In other words, we don’t need the + and - signs to tell us the “assumed” polarity of V_{AB}.

1.f) Measure E_{AB} and record its value: (*watch polarity*) E_{AB} = ____________

1.g) Measure V_{AB} and record its value: (*watch polarity*) V_{AB} = ____



In the diagram to the left, E_{AB} is the voltage of the voltage source, expressed as “the voltage at node A compared to the voltage at node B.” It’s actually unnecessary to have the + and - signs shown, since E_{AB} says it all.

NOW, V_{BA} is the voltage across resistor R1, AND once again it is a doubly-subscripted voltage, so we don’t need the + and - signs to tell us how to measure V_{BA} with a voltmeter. In other words, the + and - signs are not needed to tell us the “assumed” polarity of V_{BA}.

1.h) Measure E_{AB} and record its value: (*watch polarity*) E_{AB} = ____________

1.i) Measure V_{BA} and record its value: (*watch polarity*) V_{BA} = ____________

2. KIRCHHOFF’S VOLTAGE LAW (fondly known as KVL), With Two-Element Circuits

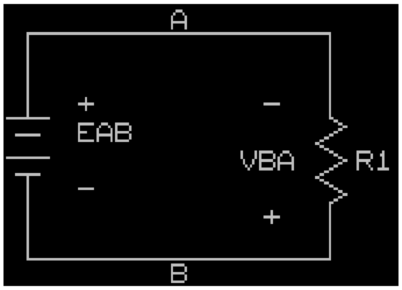
KVL is commonly stated several ways:

- the algebraic sum of the voltage drops around a closed loop equals zero
- the algebraic sum of the voltage rises around a closed loop equals zero
- the algebraic sum of the voltage rises around a closed loop equals the algebraic sum of the voltage drops around the loop

We are going to use the second statement: **the sum of the voltage rises around a loop equals zero**. Now, imagine an ant crawling from your toe to your belly button; this is a **positive rise**. Now, if that ant then crawls from your belly button to your toe, it becomes a **negative rise**. So, as we go around a loop,

if we enter a component’s voltage at its negative sign, and go through the element to its positive sign, it is considered a positive voltage rise, and recorded as positive.

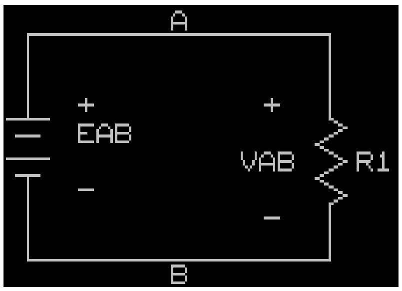
if we enter a component’s voltage at its positive sign, and go through the element to its negative sign, it is considered a negative voltage rise (or, a voltage drop), and is recorded as negative.



2.a) In the circuit to the left, we will start at point **B**, and go around the loop in a clockwise (CW) direction. Notice that we enter the negative end of each element. The KVL equation, done this way, is:

$$E_{AB} + V_{BA} = 0 \text{ volts}$$

Verify this using the measured values of E_{AB} and V_{BA} from steps 1.h and 1.i, above.



$E_{AB} + V_{BA} =$ ________ + ________ = ______________

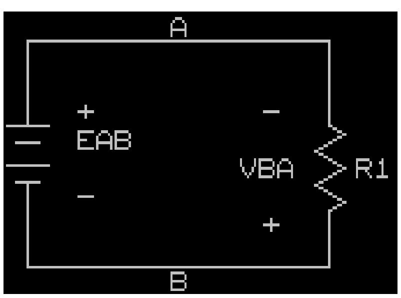
2.b) In the circuit to the left, we will start at point **B**, and go around the loop in a clockwise (CW) direction. Notice that we enter the negative end of the battery, and the positive end of the resistor. The KVL equation, done this way, is:

$$E_{AB} - V_{AB} = 0 \text{ volts}$$

Verify this using the measured values of E_{AB} and V_{AB} from steps 1.f and 1.g, above.

$E_{AB} + V_{AB} =$ ________ - ________ = ______________

OK, now that we've done two KVL loops in a clockwise (CW) direction, let's do the same two loops, starting at point B, in a counterclockwise (CCW) direction.

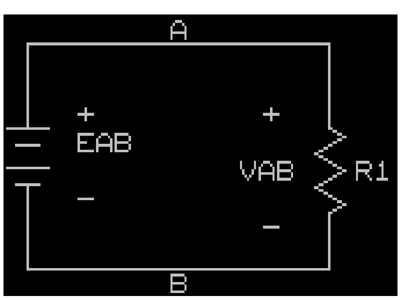


2.c) In the circuit to the left, we will start at point **B**, and go around the loop in a counterclockwise (CCW) direction. Notice that we enter the positive end of each element. The KVL equation, done this way, is:

$$-V_{BA} - E_{AB} = 0 \text{ volts}$$

Verify this using the measured values of E_{AB} and V_{BA} from steps 1.h and 1.i, above.

$-V_{BA} - E_{AB} =$ _________ - _________ = __________



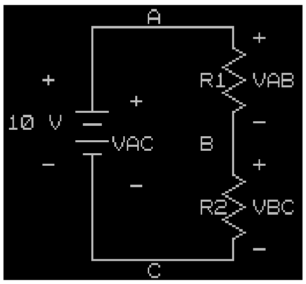
2.d) In the circuit to the left, we will start at point **B**, and go around the loop in a counterclockwise (CCW) direction. Notice that we enter the negative end of the resistor, and the positive end of the battery. The KVL equation, done this way, is:

$$V_{AB} - E_{AB} = 0 \text{ volts}$$

Verify this using the measured values of E_{AB} and V_{AB} from steps 1.f and 1.g, above.

V_{AB} - E_{AB} = ________ - ________ = _______

3. KIRCHHOFF'S VOLTAGE LAW With A Three-Element, One-Loop Circuit



3.a) In the circuit to the left, start at point **C**, and go around the loop in a clockwise (CW) direction. This is loop **CABC**. The KVL equation should be:

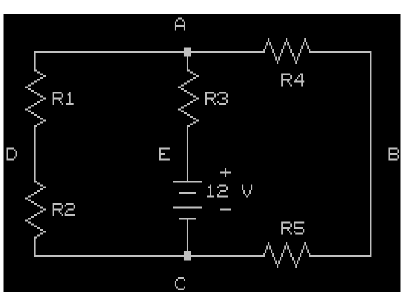
$$V_{AC} - V_{AB} - V_{BC} = 0 \text{ volts}$$

Verify that the equation above is true by measuring values of V_{AC}, V_{AB} and V_{BC}, and writing the KVL equation with measured voltage values substituted (in box below).

V_{AC} = ________ V_{AB} = ________ V_{BC} = ____________

3.b) Repeat 3.a, except start at point **C** and go CCW around the loop. This is loop **CBAC**. Write the KVL equation with measured voltage values substituted (in box below).

4. KIRCHHOFF'S VOLTAGE LAW With A Six-Element, Two-Loop Circuit



4.a) In the circuit to the left, start at point **D**, and go around the **DAECD** loop in a clockwise direction. Notice that the + and - signs for resistor polarities are not shown - recall that they are not needed. For example the voltage across resistor R1 can be expressed as V_{AD} (+ at top, - at bottom), or as V_{DA} (+ at bottom, - at top); either one is fine.

One possible KVL equation for this loop is:

$$V_{AD} + V_{EA} - 12 + V_{DC} = 0$$

Example of How To Do It: $V_{AD} + V_{EA} - 12 + V_{DC} = 0$

Write the **DAECD** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

$1.48V + 5.20V - 12V + 5.35V \stackrel{?}{=} 0 \quad 0.03V \approx 0\ V$ _

4.b) In the circuit above, start at point **D**, and go around the **DABC** loop in a clockwise direction. Write the **DABCD** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

Now write the **DABCD** loop KVL equation with measured voltage values substituted (in box below).

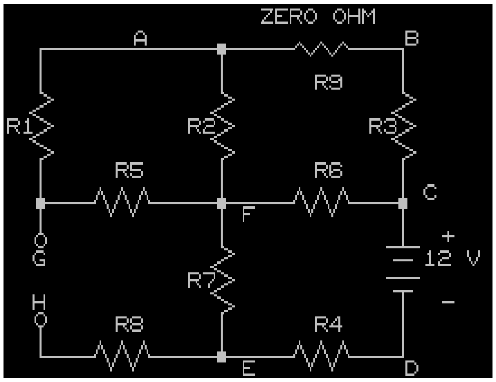
4.c) In the circuit above, start at point **C**, and go around the **CEABC** loop in a clockwise direction. Write the **CEABC** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

Now write the **CEABC** loop KVL equation with measured voltage values substituted (in box below).

At this time, get your instructor's approval of your work so far. __________

5. KIRCHHOFF'S VOLTAGE LAW With A Ten-Element, Four-Loop Circuit

Note that this circuit has R9, a zero-Ω resistor (yes, they do exist, but you should use a short wire to represent R9), and an open circuit (between nodes H and G). SUGGESTION: build it carefully, and lay it out, as drawn, so that it looks just like the schematic diagram.



5.a) In the circuit to the left, start at point **A**, and go around the **ABCDEHGA** loop in a clockwise direction.

Two possible KVL equations for this loop are:

$$V_{BA} + V_{CB} -12 + V_{ED} + V_{HE} + V_{GH} + V_{AG} = 0$$

$$V_{BA} - V_{BC} -12 + V_{ED} - V_{EH} + V_{GH} - V_{GA} = 0$$

Write the **ABCDEHGA** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

Now write the **ABCDEHGA** loop KVL equation with measured voltage values substituted (in box below).

5.b) In the circuit above, start at point **F**, and go around the **FGABCDEF** loop in a clockwise direction. Write the **FGABCDEF** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

Now write the **FGABCDEF** loop KVL equation with measured voltage values substituted (in box below).

5.c) In the circuit above, start at point **F**, and go around the **FGABCDEF** loop in a clockwise direction. Write the **FGABCDEF** loop KVL equation (that works for you) using doubly-subscripted voltages (in box below).

Now write the **FGABCDEF** loop KVL equation with measured voltage values substituted (in box below).